If Time Had No Beginning

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Work with Stav Zalel and Bruno Bento

Standard Cosmology and Time begins at the Big Bang

- The standard cosmology (ignoring inflation) implies a spacelike hypersurface in the past on which the Hubble parameter H := a-dot/a equals one in natural units. (Even with inflation, some argue that there must be such a hypersurface to the past of the inflationary era.) I will call this the Big Bang or Big Bang hypersurface.

- There (or "then") the temperature/density is Planckian and curvature is Planckian (in the timelike direction but not in spacelike directions! This is the "flatness problem")

- There is general— though not universal— agreement that the differentiable manifold structure of Lorentzian geometry is not a good description of spacetime there: a "singularity" (broadly interpreted as breakdown of known theory).

- No Lorentzian spacetime => no time => no "before" => "Time begins at the Big Bang"

- This is taken forward in quantum cosmology by the proposal that the Universe "tunnels" into existence: from "nothing" to the Big Bang hypersurface.

- The no-boundary proposal of Hartle and Hawking for a "wave function of the universe" is based on this heuristic.

Cyclic or Bouncing models

- A long standing, attractive, heuristic since ancient times
- Cyclic models can have finitely many cycles in the past or infinitely many.
- Cyclic models with no singularity: continuum time is always a good concept. No problem with "before"
- More common: cyclic models **punctuated by singularities**: a Big Bang, and then a semiclassical "epoch", and then a Big Crunch, "and then" quantum gravity, the deep quantum regime, no continuum spacetime something something….., "and then" another Big Bang, etc.
- "and then"? How to make sense of this in the absence of continuum spacetime around the Bang-Crunch events?
- To phrase the question another way: How is a cyclic model physically to be distinguished from a model in which the epochs between BB and BC all sit in spacelike relation to each other in something more akin to the "conventional" multiverse picture?

Causal Sets: a framework for exploring these issues

In the causal set approach to the problem of quantum gravity, **causal order** survives in the deep theory though continuum Lorentzian spacetime does not. The main ingredients of the approach are

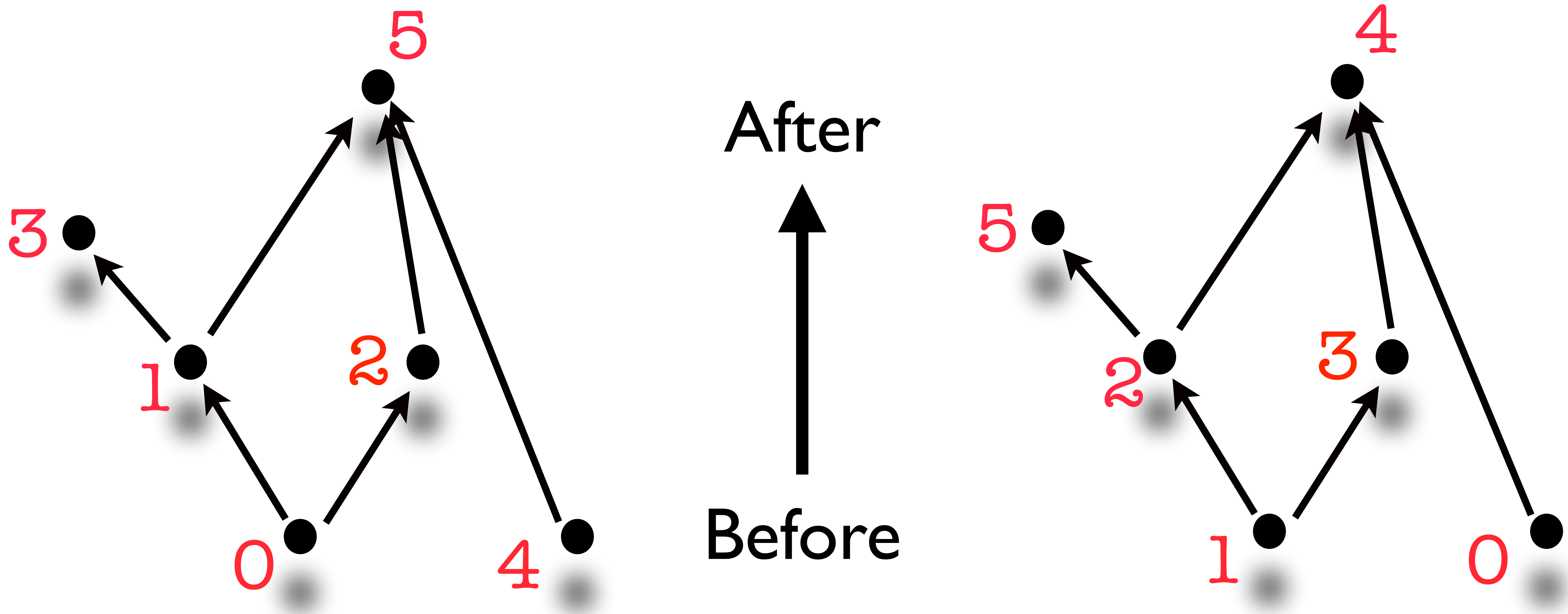
- causal order
- fundamental (spacetime) discreteness
- the path integral

Note: "causal order" is a term inherited from continuum GR. It means the order relation of "before and after" and does **not** imply "causation" as in "cause and effect" (whatever that might mean!)

That **causal order** is a more primitive notion than **time** is an ancient tradition of thought. If there is causal order in the deep theory then no matter that Lorentzian spacetime breaks down at the Big Bang, we can still understand statements that involve the phrases "and then" and "before the Big Bang" and such like.

Let's get concrete

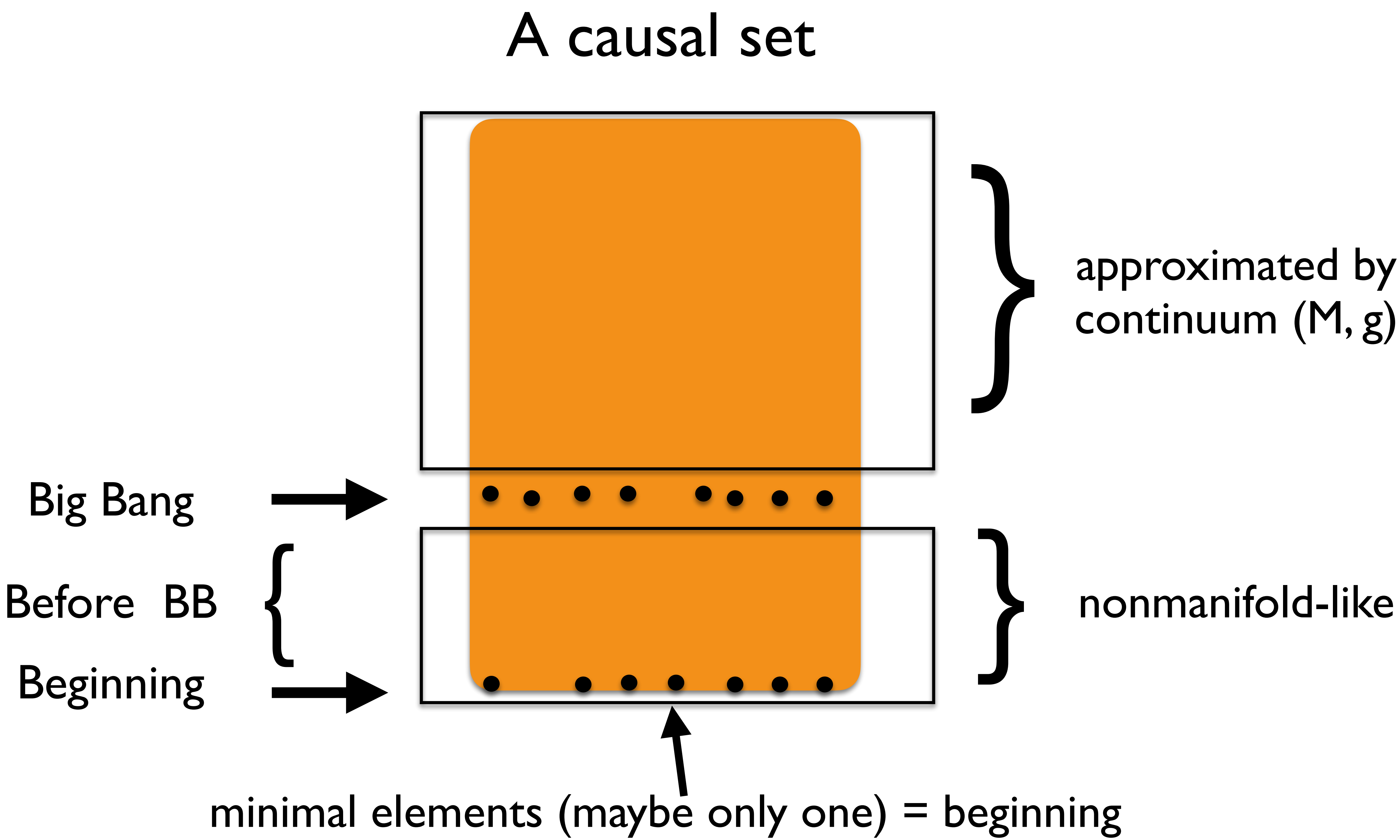
A causal set is a **discrete partial order. Hypothesis**: spacetime in GR is a continuum approximation to a causal set (our observable universe is a causal set with 10^{240} elements.



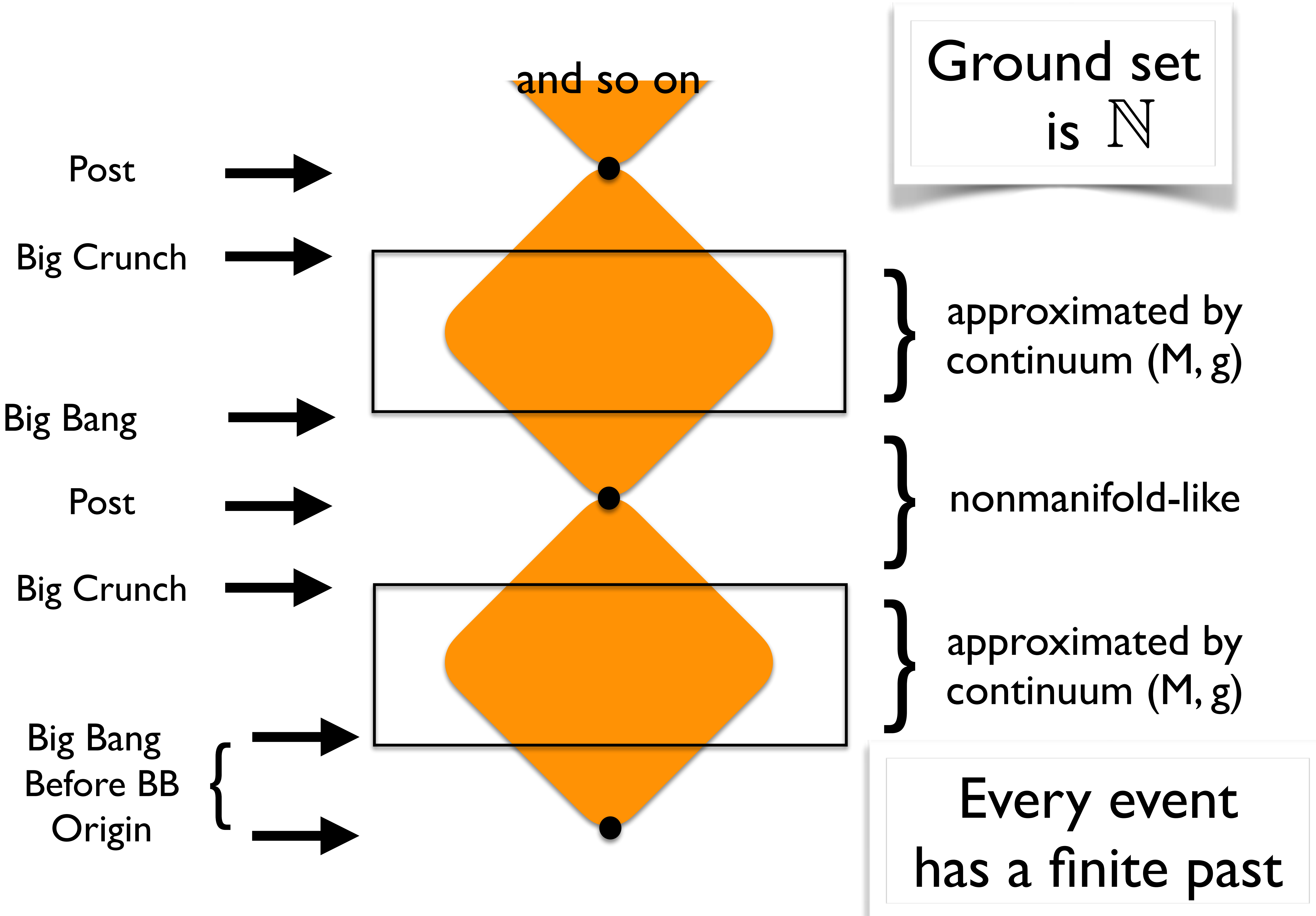
- Hasse diagram—-show only links and after is **above** before (can drop arrows)
- Locally finite
- Choose: ground set = a set of integers for concreteness
- Order relation compatible with the usual order of the integers (natural labelling)
- Adopt "General Covariance": only the causal set order is physical
- **Minimal** elements have no past, **maximal** elements have no future
- Given a continuum spacetime (M, g), there exist causal sets (C, <) which (M,g) approximates. (Strictly speaking, a conjecture for which we have much evidence.)
- Most causal sets are not **manifoldlike** in this way.

Making sense of "before" even when GR breaks down

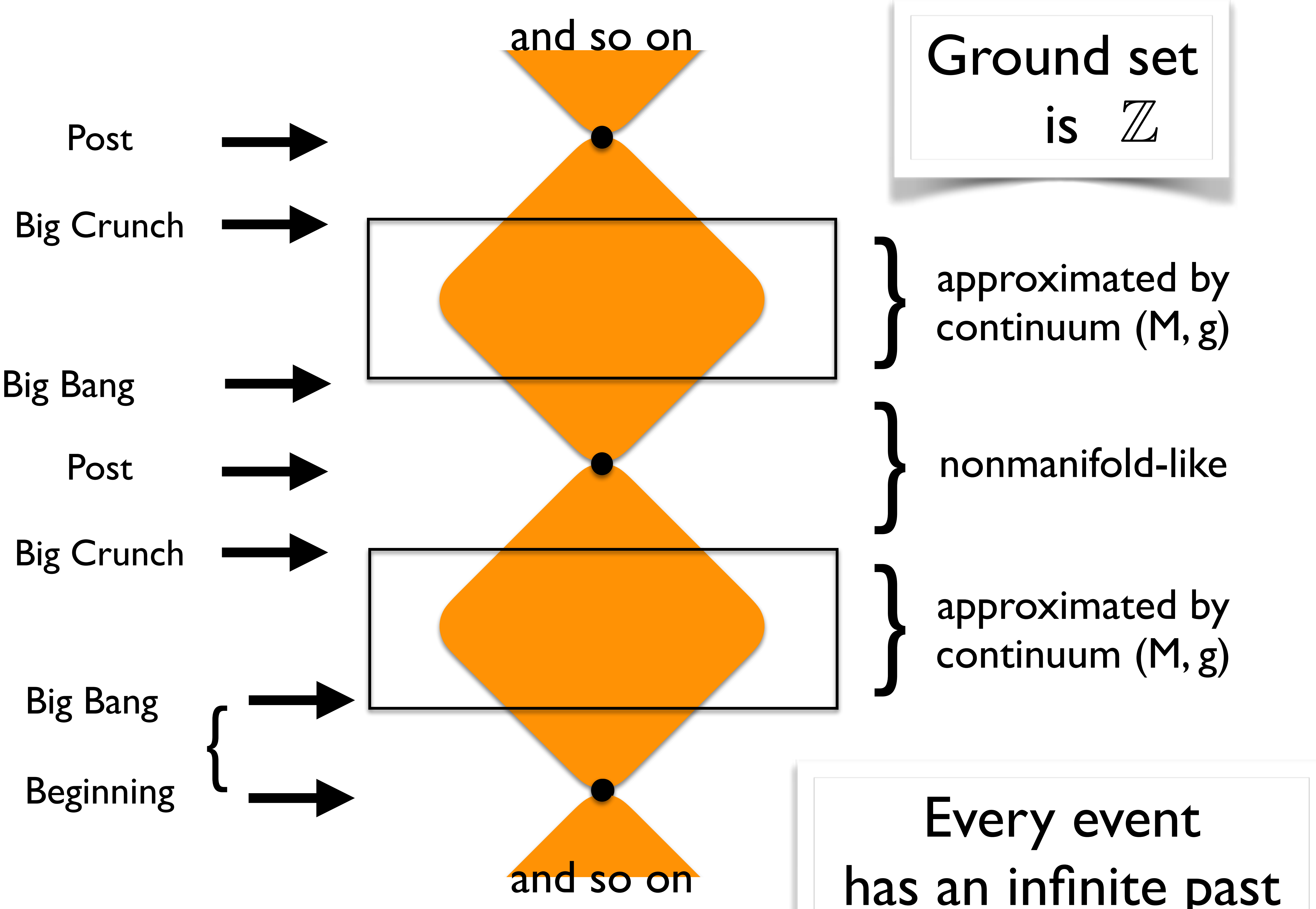
Example 1: the universe has a beginning, a Big Bang and one continuum epoch (ours)



Example 2: a bouncing universe with a beginning, many epochs



Example 3: A bouncing universe with no beginning



Summary so far: kinematics

Causal sets have the structure to accommodate

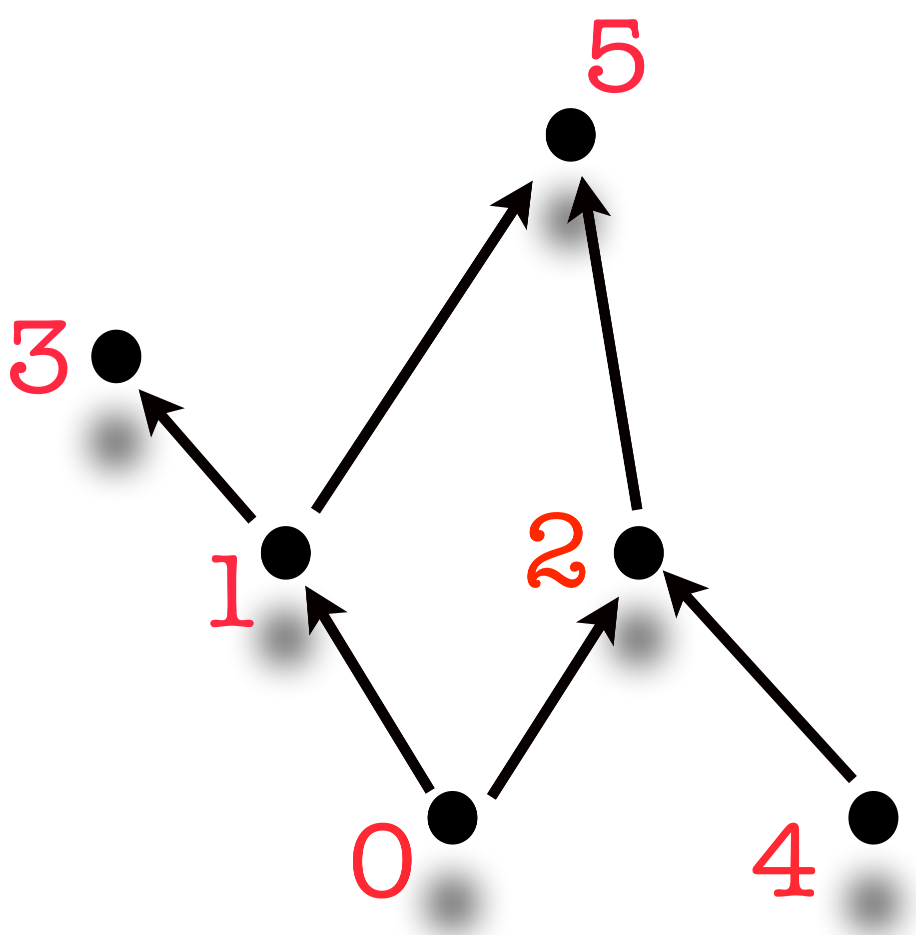
- universes with continuum regions and "deep structure" regions that resolve singularities in the continuum theory.
- universes that have a beginning (ground set is $\mathbf{N}$)
- universes that do not have a beginning (ground set is $\mathbf{Z}$)
- universes that undergo cycles of expansion and collapse—continuum-epochs punctuated by continuum-singularities
- universes where a black hole singularity (localised "Big Crunch") is followed by a new continuum-epoch or baby universe (Smolin, FD& S. Zalel)

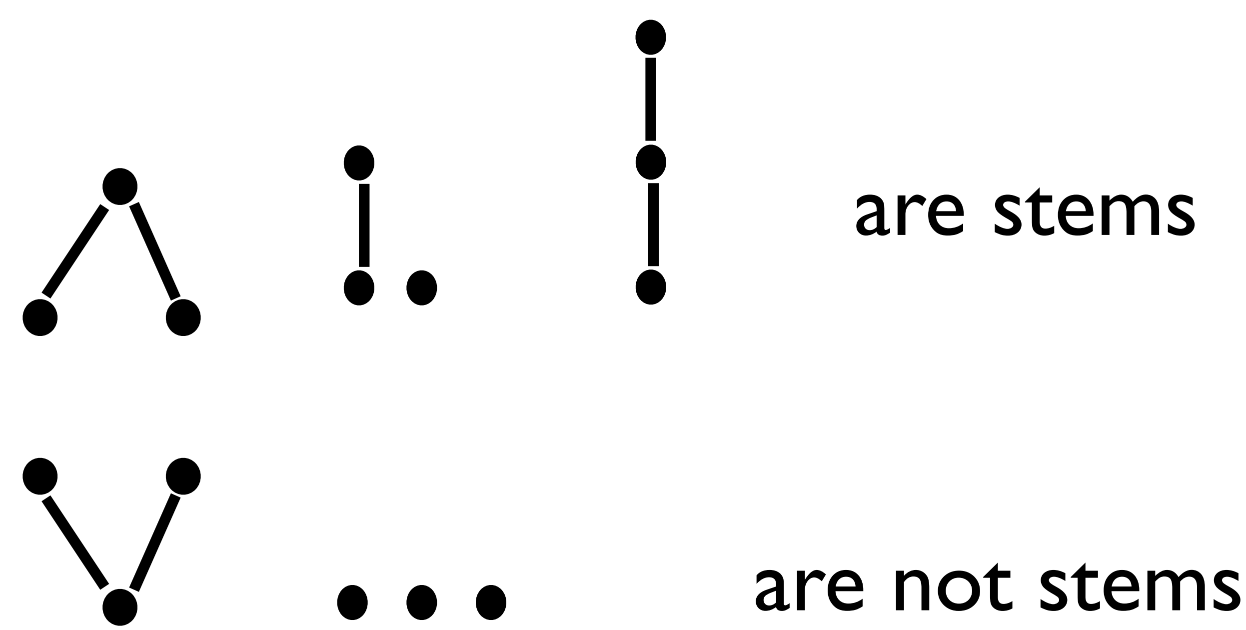
In all cases, the causal order — notion of before and after — allows sense to be made of "one thing after another"

> That's all very nice but are they physical?
> What About Dynamics?

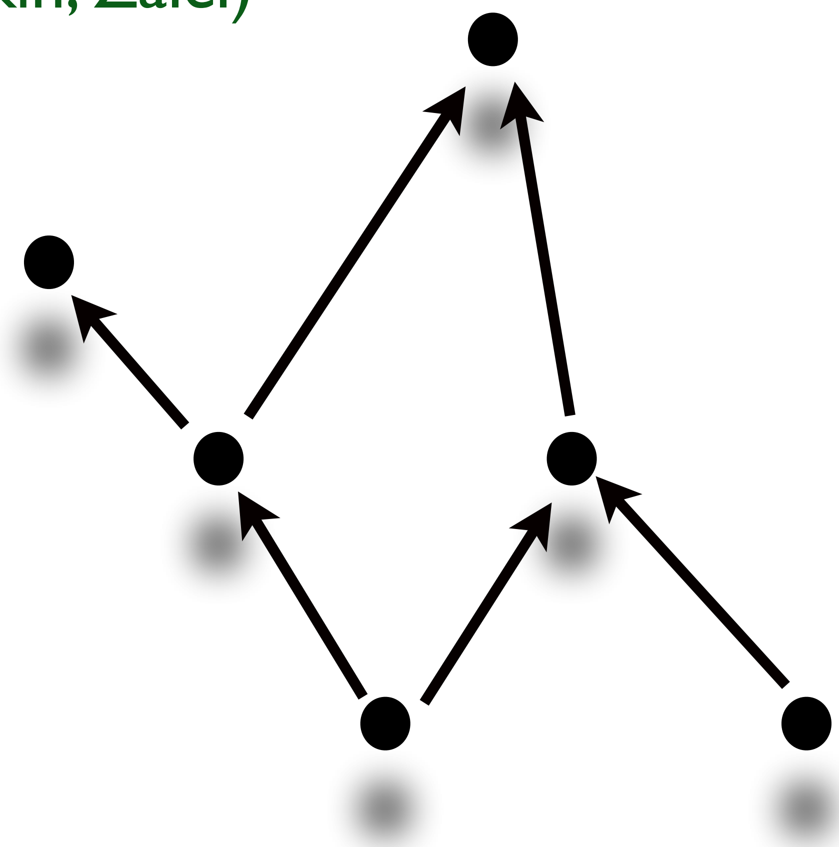
Stochastic dynamics for **past finite** causal sets: Classical Sequential Growth (Rideout&Sorkin)

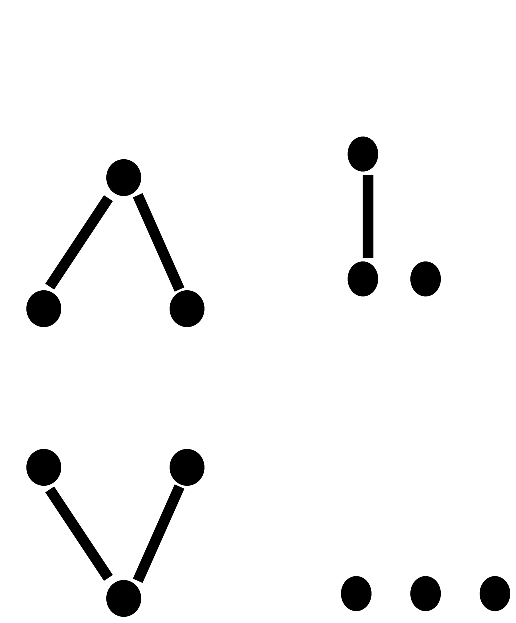
- Stochastic process grows random causal sets on ground set $\mathbb{N}$: there's a beginning, by fiat
- Dynamics respects Discrete General Covariance and Bell Causality
- Simplest case: Transitive Percolation (TP) ("Lorentzian Erdos-Renyi Random Graph")
- With probability one, TP grows a bouncing universe with infinitely many posts (but probably not manifoldlike epochs)
- Statements about **stems** are covariant (label independent), measurable events
- **Theorem:** (solution of the "problem of covariance" in CSG) **all** covariant statements in the event algebra are about stems (up to sets of measure zero) (Brightwell, FD, Garcia, Henson&Sorkin)





Dynamics for past finite causal sets go covariant (FD, Imambaccus, Owens, Sorkin, Zalel)





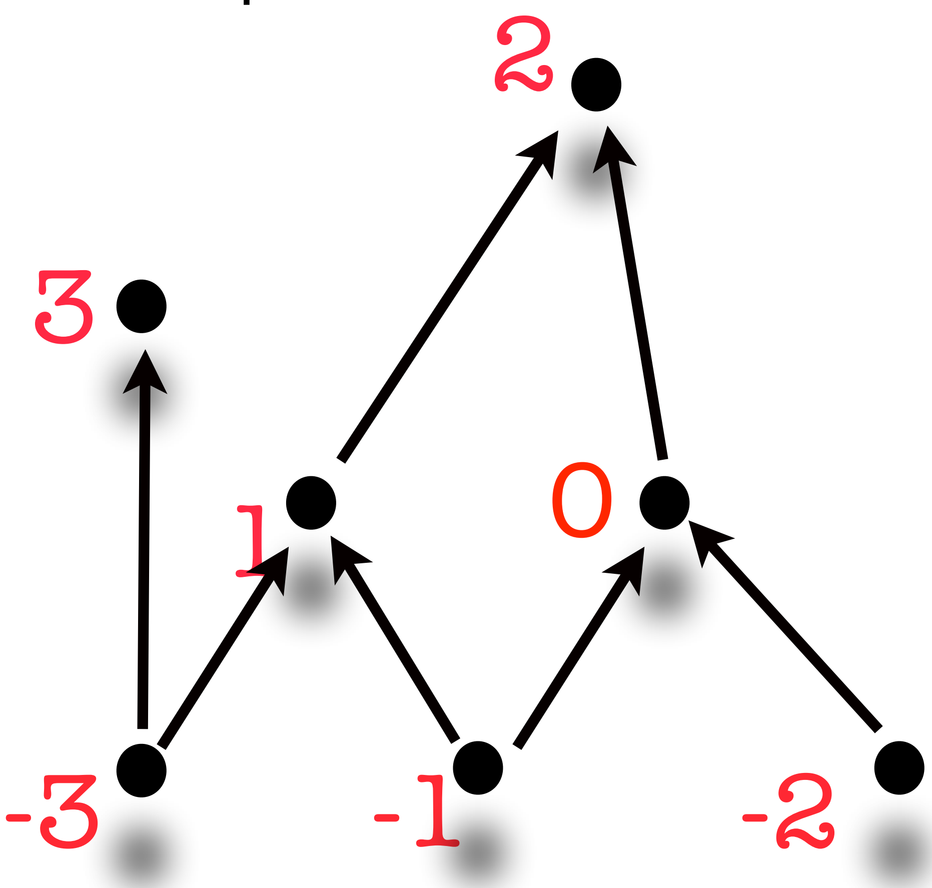
are 3-stems

are not 3-stems

- For each past finite causal set C, and each n there is a set of its n-stems
- Consider a tree, the nodes of which are the possible sets of n-stems of a causal set C: **"covtree"** arranged in levels labelled by n
- Consider a dynamics as a random walk up covtree: at stage n the walk chooses a set of n-stems, and at the next stage a set of (n+1)-stems etc.
- Never introduce a labelling
- **Theorem:** Each path up covtree corresponds to at least one causal set. So a random walk up covtree is a possible causal set dynamics.
- In this framework, one only ever has pure, covariant thoughts about physical properties: stems
- Can we understand Bell Causality in this setting??

Dynamics for past **infinite** causal sets (B. Bento, FD, S. Zalel,)

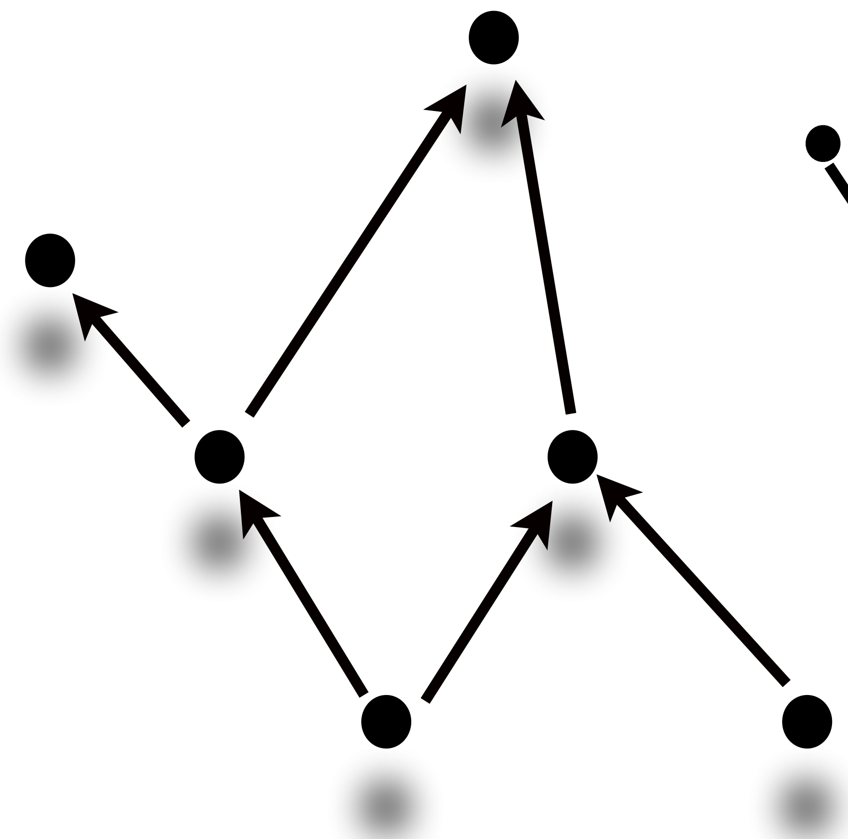
- Seek a stochastic process that grows causal sets on ground set $\mathbb{Z}$**:** there may or may not be a beginning
- Proposal: **alternating** growth dynamics where add an element to future and then to the past and so on.

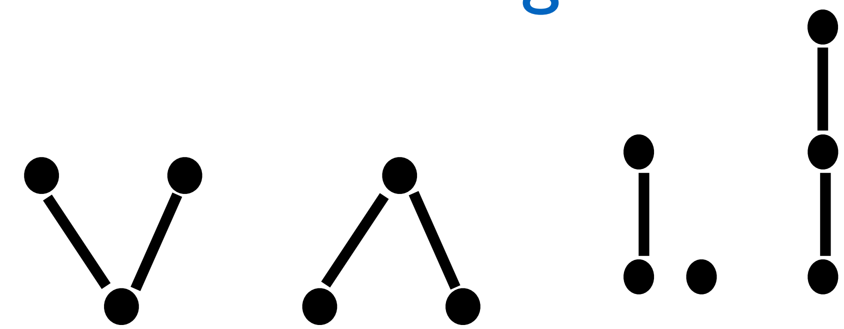


- Lose the potential for the identification of the growth process and passage of time
- **Theorem:** Sample space is: (i) past finite with infinite minimal elements (ii) future finite with infinite maximal elements or (iii) past and future infinite
- Becomes hard to define a condition of Bell Causality
- Thus, no analogue of derivation of CSG models
- **IF** just adopt alternating CSG probabilities, then we find that alt-CSG satisfies Discrete General Covariance iff it is Transitive Percolation
- In Alt-TP, with probability one, get a **past and future infinite bouncing universe.**

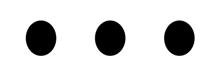
- No stems in Alt-TP! What are (localised) covariant properties?
- Statements about **convex-sets** are covariant, measurable events
- **Theorem:** in Alt-TP every convex-set has probability one. No discrimination possible!
- **The beginning is an anchor**. Without it, covariant statements are less useful.

Dynamics for **past infinite** causal sets go covariant (Bento, FD, Zalel)





are 3-convex-sets



is not a 3-convex-set

- For each causal set C on Z, and each n there is a set of its n-convex-sets
- Consider a tree, the nodes of which are the possible sets of n-convex-sets of a causal set C: **"convex-covtree"** arranged in levels labelled by n
- Consider a dynamics as a random walk up covtree: at stage n the walk chooses a set of n-convex-sets , and at the next stage a set of (n+1)-convex-sets etc.
- Never introduce a labelling
- **Theorem:** Each path up convex-covtree corresponds to at least one causal set.
- **Theorem:** Some paths up convex-covtree have a maximal element: universe is **finite**!
- **Theorem:** Paths in convex-covtree exist corresponding to causal sets on Z, N and N-minus.
- There's a subtree which restricts to paths corresponding only to Z causal sets
- Can we find a family of physically motivated/interesting such models (Transitive percolation won't be one of them)?

Dynamics for **past infinite** causal sets: viable or not?

- Confession: I would like to kill off past infinite causal sets, show they can't produce useful cosmologies. Personally, I abhor a physical infinity.
- Transitive Percolation is an important model in the CSG framework: cosmic renormalisation and Sorkin's argument (in Indications of Causal Set Cosmology) that the CSG framework contains the possibility of a self-tuning "Tolman-Boltzmann" universe of larger and larger cycles.
- If including past infinite causal sets means TP has to be abandoned because it has no quasi-local covariant "observables" in its past infinite guise then we stand to lose a lot.
- Interesting structure though — convex-covtree and Z-covtree are nice to think about.

- **What about the quantum?**
- No one (?) thinks the universe is a single causal set
- Classical stochastic models are a warm up for the quantum case
- In a path integral framework, the dynamics is given by a sum over causal sets. The question becomes: what causal sets are included? Past finite or past infinite?
- The interpretation of this path integral framework is a work in progress, but the concept of "event" is the same as used here. The question is the same: can convex-set events make sense and be useful in the dynamical model or only stem events—which are anchored to a beginning?

Thank you for listening